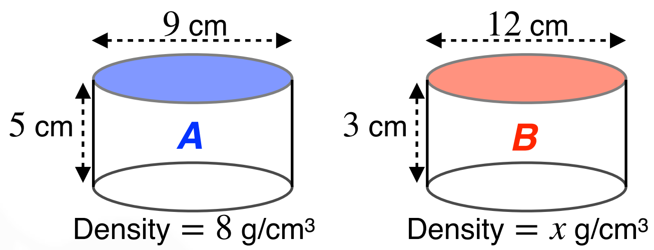
Pythagoras' Theorem

Starter

1. **(Review of last lesson)**
The two solid cylinders shown have the same mass. Calculate the density, x g/cm³, of cylinder B.



Working: **A**: $r = 4.5, h = 5$

Volume, $V = \pi r^2 h$: $V = \pi \times 4.5^2 \times 5$
$= 101.25\pi$

Density $= \dfrac{\text{Mass}}{\text{Volume}}$: $8 = \dfrac{\text{Mass}}{101.25\pi}$

Mass $= 8 \times 101.25\pi = 810\pi$

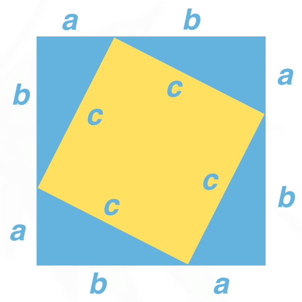
B: $r = 6, h = 3$

Volume, $V = \pi r^2 h$: $V = \pi \times 6^2 \times 3$
$= 108\pi$

Density $= \dfrac{\text{Mass}}{\text{Volume}}$: Density $= \dfrac{810\pi}{108\pi} = 7.5$

The density of cylinder B is 7.5 g/cm³.

E.g. 1 By finding the area of the large square in two different ways, prove Pythagoras Theorem i.e. that $a^2 + b^2 = c^2$



Working: Length of one side of big square $= a + b$

Area of large square $= (a+b)^2$
$= (a+b)(a+b)$
$= a^2 + ab + ab + b^2$
$= a^2 + 2ab + b^2$

Also, area of large square = area of small square + area of 4 triangles

$$= c^2 + 4 \times \left(\frac{a \times b}{2}\right)$$
$$= c^2 + 2ab$$

Equating the two formulae for the area of the big area:

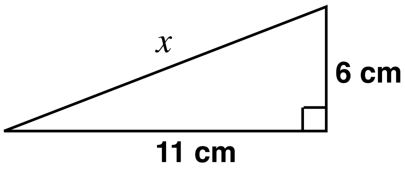
$$a^2 + 2ab + b^2 = c^2 + 2ab$$

Subtracting $2ab$ from both sides: $a^2 + b^2 = c^2$

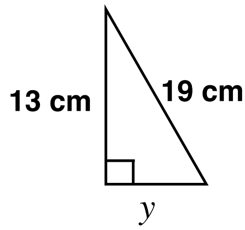
N.B. Always label the hypotenuse — this value is on its own on one side of the formula.

E.g. 2 Find the length of the missing side of these right-angled triangles:

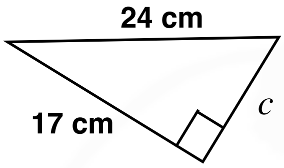
(a)



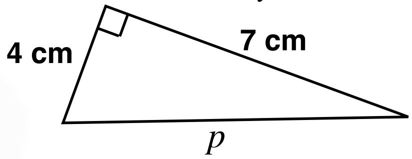
(b)



(c)



(d)



Working: (a) ***Label the hypotenuse***

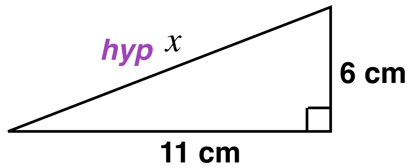
$a^2 + b^2 = c^2$

$6^2 + 11^2 = x^2$

$36 + 121 = x^2$

$x^2 = 157$

$x = \sqrt{157} = 12.5$ (3 s.f.)



(b) ***Label the hypotenuse***

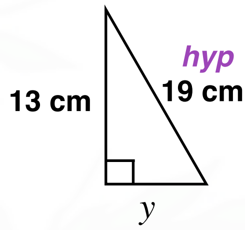
$a^2 + b^2 = c^2$

$13^2 + y^2 = 19^2$

$y^2 = 19^2 - 13^2$

$y^2 = 192$

$y = \sqrt{192} = 13.9$ (3 s.f.)



(c) ***Label the hypotenuse***

Even though the side is labelled as c,

it is not hypotenuse.

Be careful when substituting into the formula.

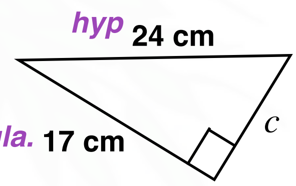
$a^2 + b^2 = c^2$

$c^2 + 17^2 = 24^2$

$c^2 = 24^2 - 17^2$

$c^2 = 287$

$y = \sqrt{192} = 16.9$ (3 s.f.)



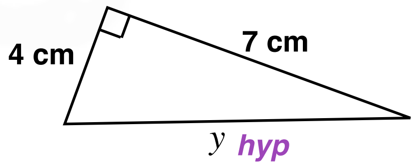
(d) ***Label the hypotenuse***

$a^2 + b^2 = c^2$

$4^2 + 7^2 = p^2$

$y^2 = 65$

$y = \sqrt{65} = 8.06$ (3 s.f.)



E.g. 3 Find the distance between the points $(-3, 7)$ and $(8, -1)$.

Working: Horizontal distance between the points $= 8 - -3 = 11$
Vertical distance between the points $= -1 - 7 = -8$
Let x be the distance between the points
$x^2 = 11^2 + (-8)^2 \quad \Rightarrow \quad x^2 = 185 \quad \Rightarrow \quad x = \sqrt{185}$
The distance between the points is $\sqrt{185} = 13.6$ cm (3 s.f.)

E.g. 4 Determine whether a triangle with sides 19 cm, 16 cm, 10 cm has a right angle.

Working: If the triangle has a right-angle the lengths will satisfy Pythagoras' theorem.
$a^2 + b^2 = c^2$: LHS: $10^2 + 16^2 = 100 + 256 = 356$
RHS: $19^2 = 361$
Since $356 \neq 361$, the lengths do not satisfy Pythagoras' theorem and so the triangle does not have a right angle.

E.g. 5 The lengths in a right-angled triangle are x, $3x$ and 45 with 45 cm being the hypotenuse. Find x.

Working: $a^2 + b^2 = c^2$:
$x^2 + (3x)^2 = 45^2$ ***use brackets***
$x^2 + 9x^2 = 2025$ ***the 3 and the x get squared***
$10x^2 = 2025$
$x^2 = 202.5$
$x = \sqrt{202.5} = 14.2$ cm (3 s.f.)

Video: **Pythagoras' Theorem**

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p315 M10.5 Qu 1-8; p317 M10.6 Qu 1-18
A*-G class textbook:	p278 M10.5 Qu 1-6; p280 M10.6 Qu 1-20 even
9-1 homework book:	p107 M10.5 Qu 1-8; p108 M10.6 Qu 1-12
A*-G homework book:	p78 M10.5 Qu 1-5; p79 M10.6 Qu 1-12