Simplifying surds

Starter

1. **(Review of last lesson)** Find the acute angle between the two diagonals of a rectangle whose sides are 6 cm and 9 cm.

Working: The required angle is $2 \times \theta$

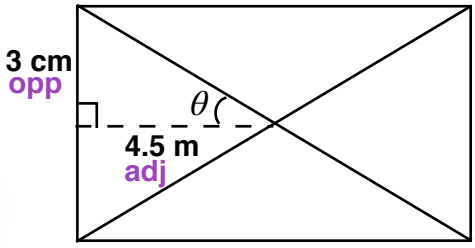
opp and adj $\Rightarrow$ tan

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$: $\tan\theta = \dfrac{3}{4.5}$

$\theta = \tan^{-1}\dfrac{3}{4.5}$

$2\theta = 2\tan^{-1}\dfrac{3}{4.5}$

$= 67.4°$



The acute angle between the two diagonals is $67.4°$ (3 s.f.).

2. (a) Write down the first ten square numbers.

(b) Hence simplify: (i) $\sqrt{36}$ (ii) $\sqrt{81}$ (iii) $\sqrt{32}$

Working: (a) $1, 4, 9, 16, 25, 36, 49, 64, 81, 100$

(b) (i) $\sqrt{36} = 6$

(ii) $\sqrt{81} = 9$

(iii) $\sqrt{32} - 32$ is not a square number. See below for the method.

E.g. Simplify: (a) $\sqrt{20}$ (b) $\sqrt{32}$

Working: (a) $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$

(b) *32 has two square numbers, 16 and 4, which are factors. The best one to choose is 16.*

$\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$

If 4 is chosen, the simplification requires two bites at the cherry.

$\sqrt{32} = \sqrt{4 \times 8} = \sqrt{4} \times \sqrt{8} = 2\sqrt{8} = 2\sqrt{4 \times 2} = 2\sqrt{4} \times \sqrt{2} = 4\sqrt{2}$

E.g. 1 Without a calculator, simplify these surds:

(a) $\sqrt{12}$ (b) $\sqrt{50}$ (c) $\sqrt{48}$ (d) $\sqrt{200}$

Working: (a) $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

(b) $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

(c) $\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

...or...

$\sqrt{48} = \sqrt{4 \times 12} = \sqrt{4} \times \sqrt{12} = 2\sqrt{12} = 2\sqrt{4 \times 3} = 2\sqrt{4} \times \sqrt{3} = 4\sqrt{3}$

(d) $\sqrt{200} = \sqrt{100 \times 2} = \sqrt{100} \times \sqrt{2} = 10\sqrt{2}$

...or...

$\sqrt{200} = \sqrt{25 \times 8} = \sqrt{25} \times \sqrt{8} = 5\sqrt{8} = 5\sqrt{4 \times 2} = 5\sqrt{4} \times \sqrt{2} = 10\sqrt{2}$

E.g. 2 Without a calculator, simplify: (a) $\sqrt{8} \times \sqrt{2}$ (b) $\dfrac{\sqrt{27}}{\sqrt{12}}$

(c) $\sqrt{15} \times \sqrt{3}$ (d) $\dfrac{\sqrt{240}}{\sqrt{6}}$

Working: (a) $\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$

(b) $\dfrac{\sqrt{27}}{\sqrt{12}} = \sqrt{\dfrac{27}{12}} = \sqrt{\dfrac{9}{4}} = \dfrac{3}{2}$

(c) $\sqrt{15} \times \sqrt{3} = \sqrt{45} = \sqrt{9 \times 5} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$

(d) $\dfrac{\sqrt{240}}{\sqrt{6}} = \sqrt{\dfrac{240}{6}} = \sqrt{40} = \sqrt{4 \times 10} = \sqrt{4} \times \sqrt{10} = 2\sqrt{10}$

Adding and subtracting surds

E.g. 3 Given that $2x + 3x = 5x$, write down, in surd form, the value of $2\sqrt{7} + 3\sqrt{7}$.

Working: $2\sqrt{7} + 3\sqrt{7} = 5\sqrt{7}$

E.g. 4 Without a calculator, simplify, where possible:

(a) $7\sqrt{5} - 3\sqrt{5}$ (b) $\sqrt{32} + 3\sqrt{2}$

(c) $2\sqrt{27} + 6\sqrt{18}$ (d) $5\sqrt{27} + 3\sqrt{28}$

Working: (a) $7\sqrt{5} - 3\sqrt{5} = 4\sqrt{5}$

(b) $\sqrt{32} + 3\sqrt{2} = \sqrt{16 \times 2} + 3\sqrt{2} = 4\sqrt{2} + 3\sqrt{2} = 7\sqrt{2}$

(c) $\begin{aligned} 2\sqrt{27} + 6\sqrt{18} &= 2\sqrt{9 \times 3} + 6\sqrt{9 \times 2} \\ &= 2\sqrt{9} \times \sqrt{3} + 6\sqrt{9} \times \sqrt{2} \\ &= 2 \times 3 \times \sqrt{3} + 6 \times 3 \times \sqrt{2} \\ &= 6 \times \sqrt{3} + 18 \times \sqrt{2} \\ &= 24 \times \sqrt{3} \end{aligned}$

(d) $5\sqrt{27} + 3\sqrt{28}$ – cannot be simplified since 27 and 28 do not have any common factors

Video: Surds

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p12 E1.2 Qu 1ace…, 2ace…, 3-5, 6ace…, 7, 8ace…, 9
A*-G class textbook:	p12 E1.2 Qu Qu 1ace…, 2ace…, 3, 4, 5ace…, 6, 7ace…,
9-1 homework book:	p4 E1.2 Qu 1ace…, 2, 3ace…, 4ace…, 5, 6
A*-G homework book:	p4 E1.2 Qu 1ace…, 2ace…, 3, 4ace…